Functions

Parity of a function

Parity
- Even
  - 1 Domain is centered at O
- Odd
  - 1 Domain is centered at O

Parity of a function

$x \in D_f \; ; -x = opp(x) \in D_f$

- Parity
  - Even
    - 1 Domain is centered at O
  - Odd
    - 1 Domain is centered at O

<u>Centered at O</u>

-]-∞;+∞[
-]-1;1[or [-1;1]
-]-∞;-1[∪]1;+∞[
-]-∞;-1]∪[1;+∞[

<u>Not centered at O</u>

-]-∞;1[or]-∞;1]
-]-1;+∞[or [-1;+∞[
- [-1;1[or]-1;1]
-]-∞;-1[∪[1;+∞[
-]-∞;-2]∪[3;+∞[

Parity of a function

- Parity
  - Even
    - ① Domain is centered at O
    - ② $f(-x) = f(x)$
  - Odd
    - ① Domain is centered at O
    - ② $f(-x) = -f(x)$

Example:

❶ $f(x) = x^2 + |x|$

D=IR is centered at O

$$f(-x) = (-x)^2 + |-x|$$
$$= x^2 + |x|$$
$$= f(x)$$

So f is even function

Parity of a function

Parity

- Even
  - ① Domain is centered at O
  - ② $f(-x) = f(x)$
- Odd
  - ① Domain is centered at O
  - ② $f(-x) = -f(x)$

Example:

❷ $f(x) = \frac{1}{x}$

D=]-∞;0[∪]0;+∞[is centered at O

$$f(-x) = \frac{1}{-x} = -\frac{1}{x}$$

$$= -f(x)$$

So f is odd function

Parity of a function (Application)

Study the parity of the function f in each case.

❶ $f(x) = x^2 + 3 \quad D = IR$

1. Domain is IR=]-∞;+∞[: centered at O
2. $f(-x) = (-x)^2 + 3 = x^2 + 3 = f(x)$ so f is even

❷ $f(x) = x^3 - x \quad D = IR$

1. Domain is IR=]-∞;+∞[: centered at O
2. $f(-x) = (-x)^3 - (-x) = -x^3 + x \neq f(x)$
   $f(-x) = -(x^3 - x) = -f(x)$ so f is odd

Parity of a function (Application)

Study the parity of the function f in each case.

❸ $f(x) = x^4 + 2x^2 \quad D =]-3;4[$

Domain is not centered at O so f is not even nor odd

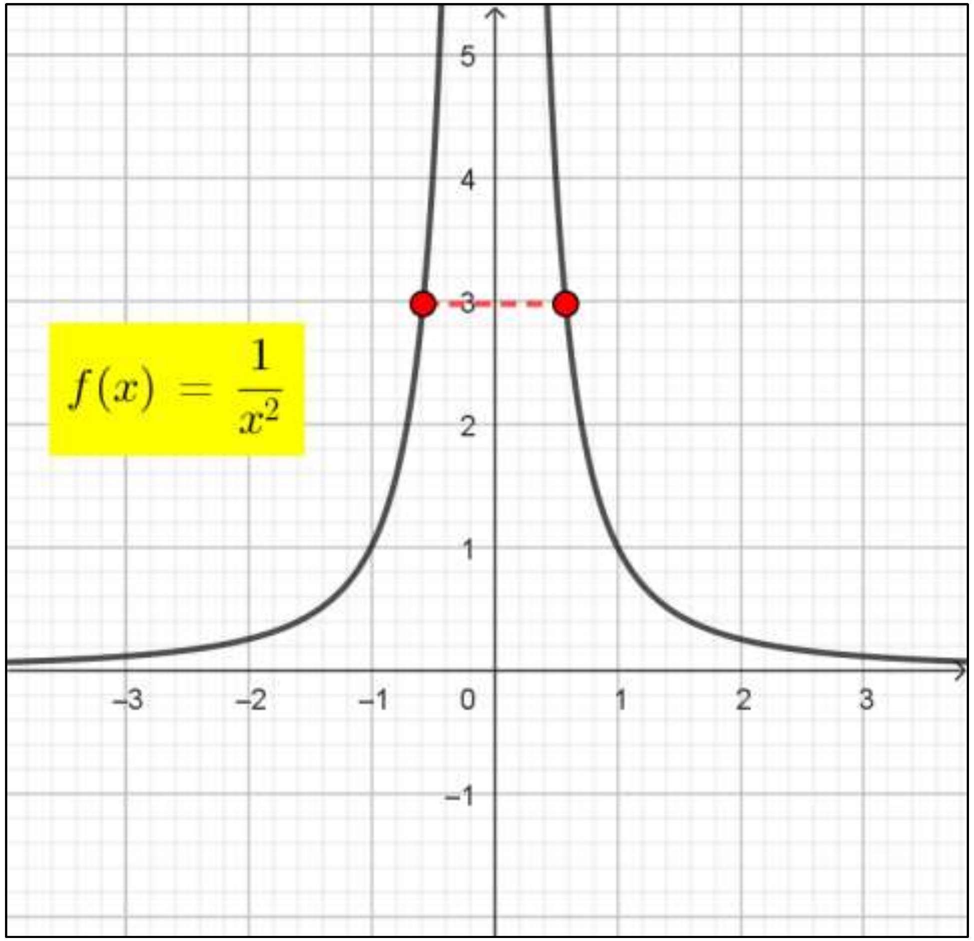
❹ $f(x) = x^4 + 3x - 4 \quad D = IR$

1. Domain is centered at O
2. $f(-x) = (-x)^4 + 3(-x) - 4 = x^4 - 3x - 4 \neq f(x)$
   $f(-x) = -(-x^4 + 3x + 4) \neq -f(x)$

So f is not even nor odd.

Parity of a function (Graphical interpretation)

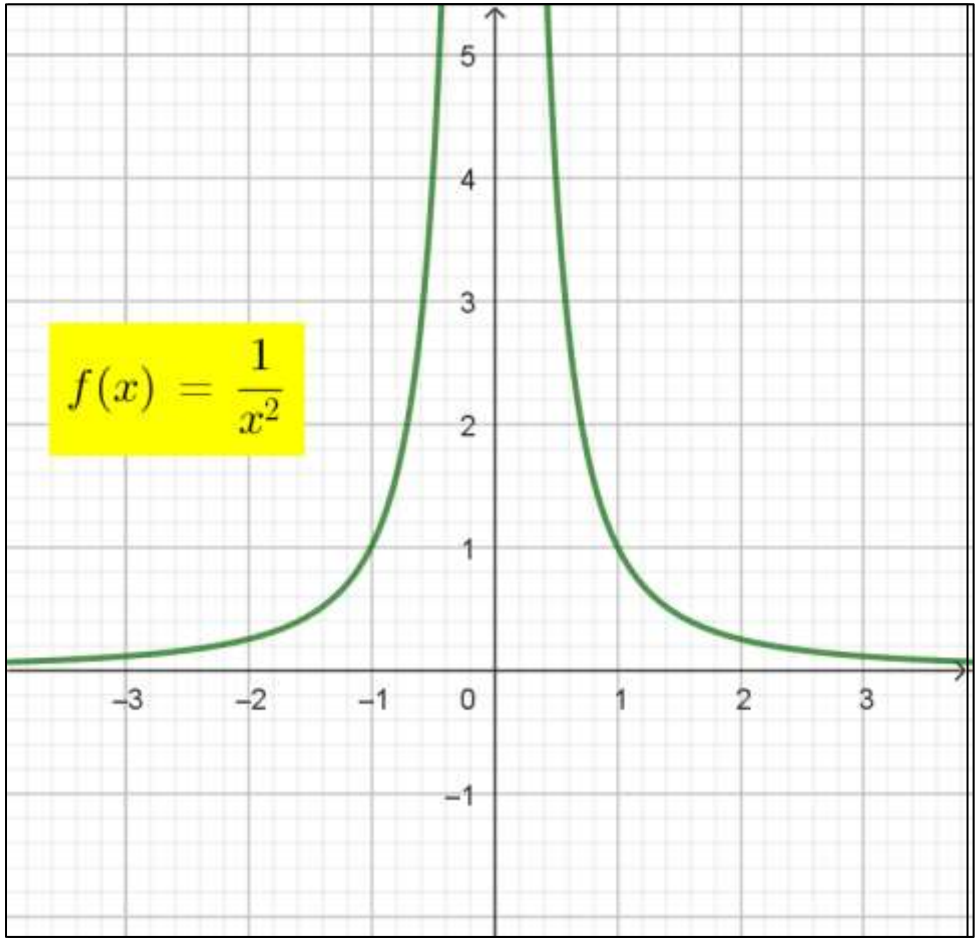
Even function



The two points of abscissas x and $-x$ are symmetric with respect to (y'y)

Parity of a function (Graphical interpretation)

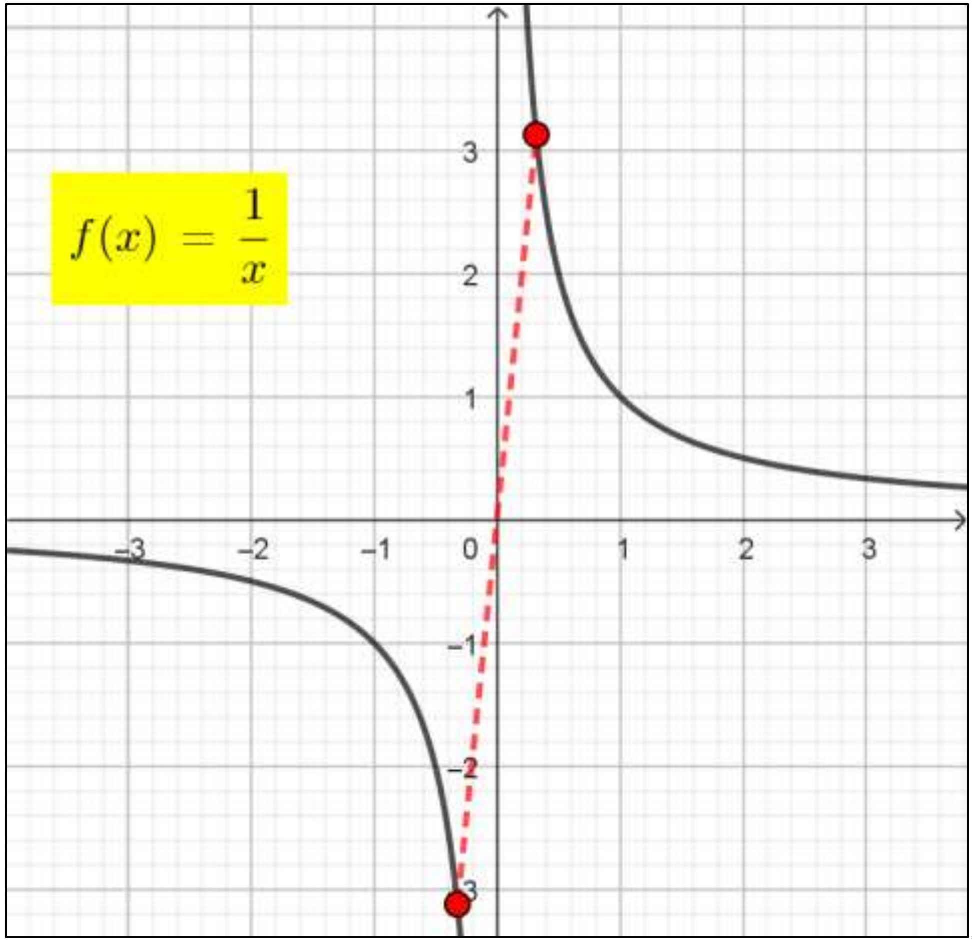
Even function



If f is an even function:
(y'y) is an axis of symmetry

Parity of a function (Graphical interpretation)

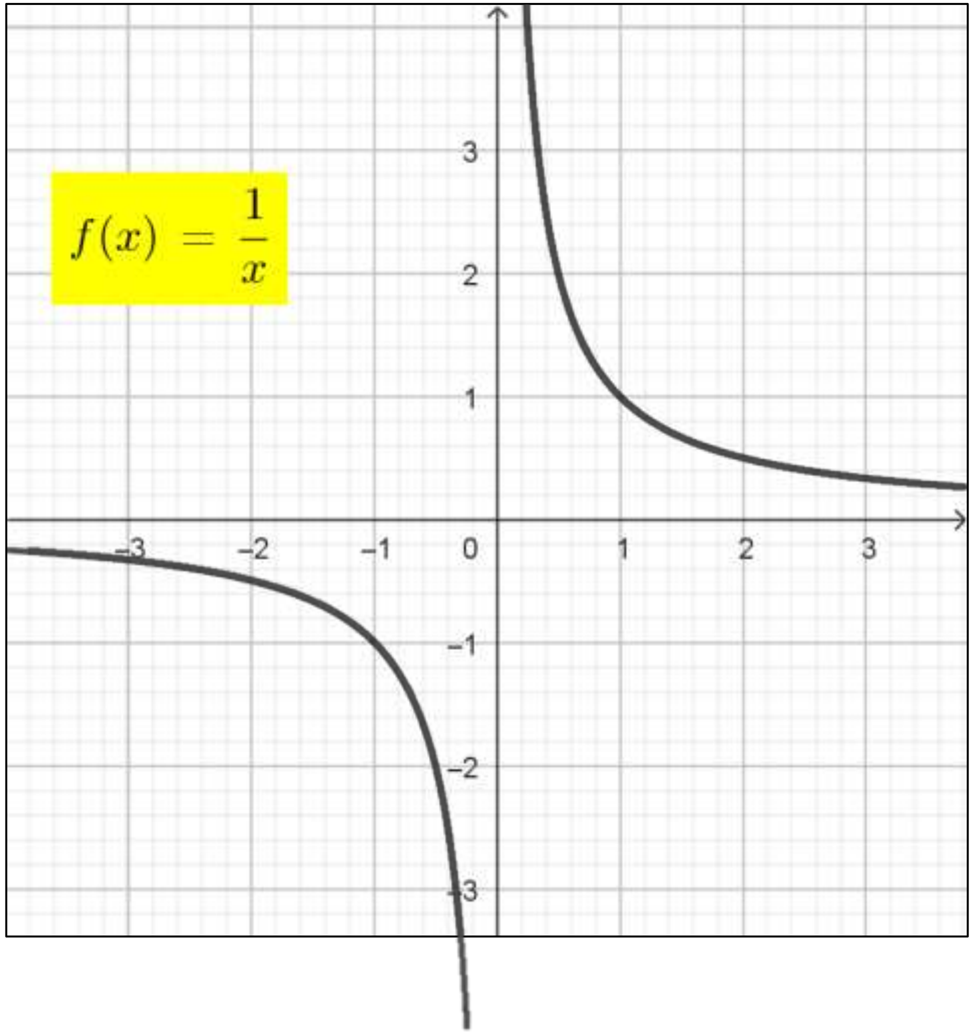
Odd function



The two points of abscissas x and $-x$ are symmetric with respect to the origin O

Parity of a function (Graphical interpretation)

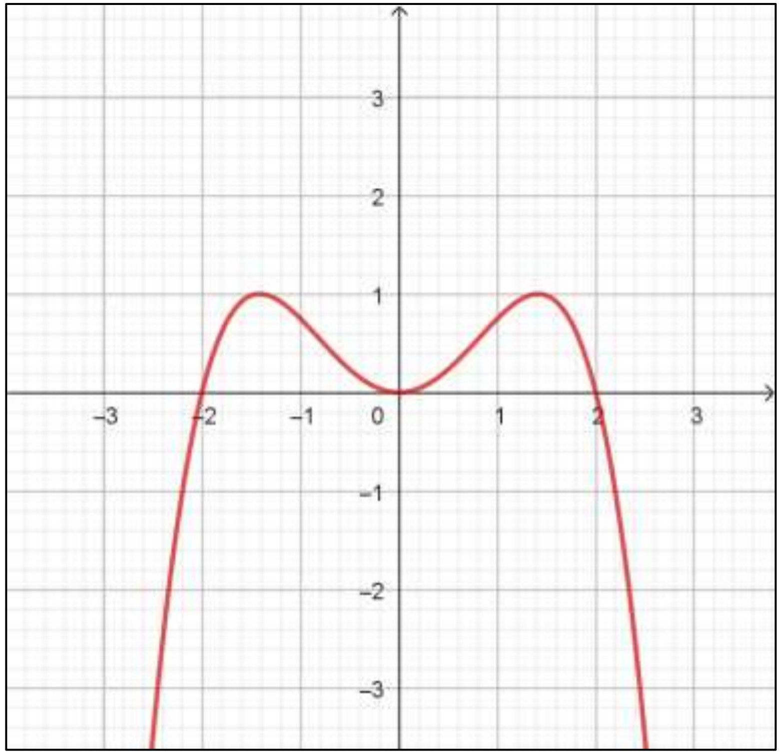
Odd function



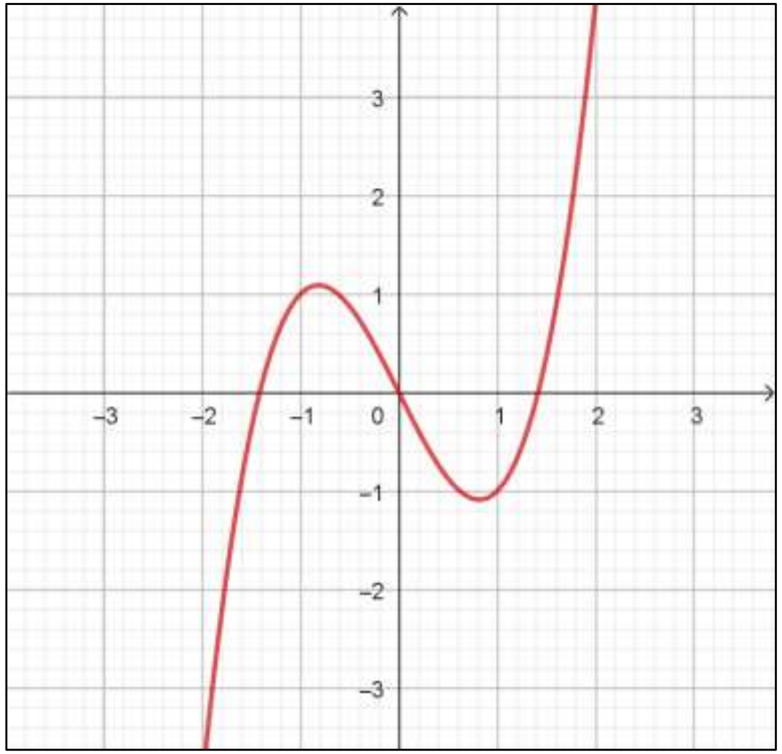
If f is an odd function: O(0,0) is a point of symmetry

Parity of a function (Application)

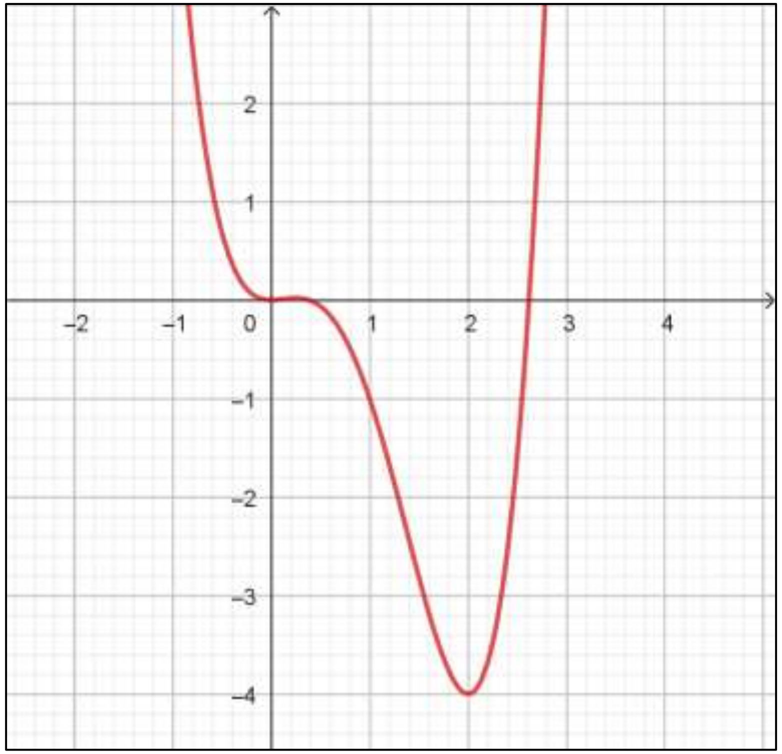
Study the parity of the function f in each case.



Even function:
Domain is centered at O
(y'y) is an axis of symmetry

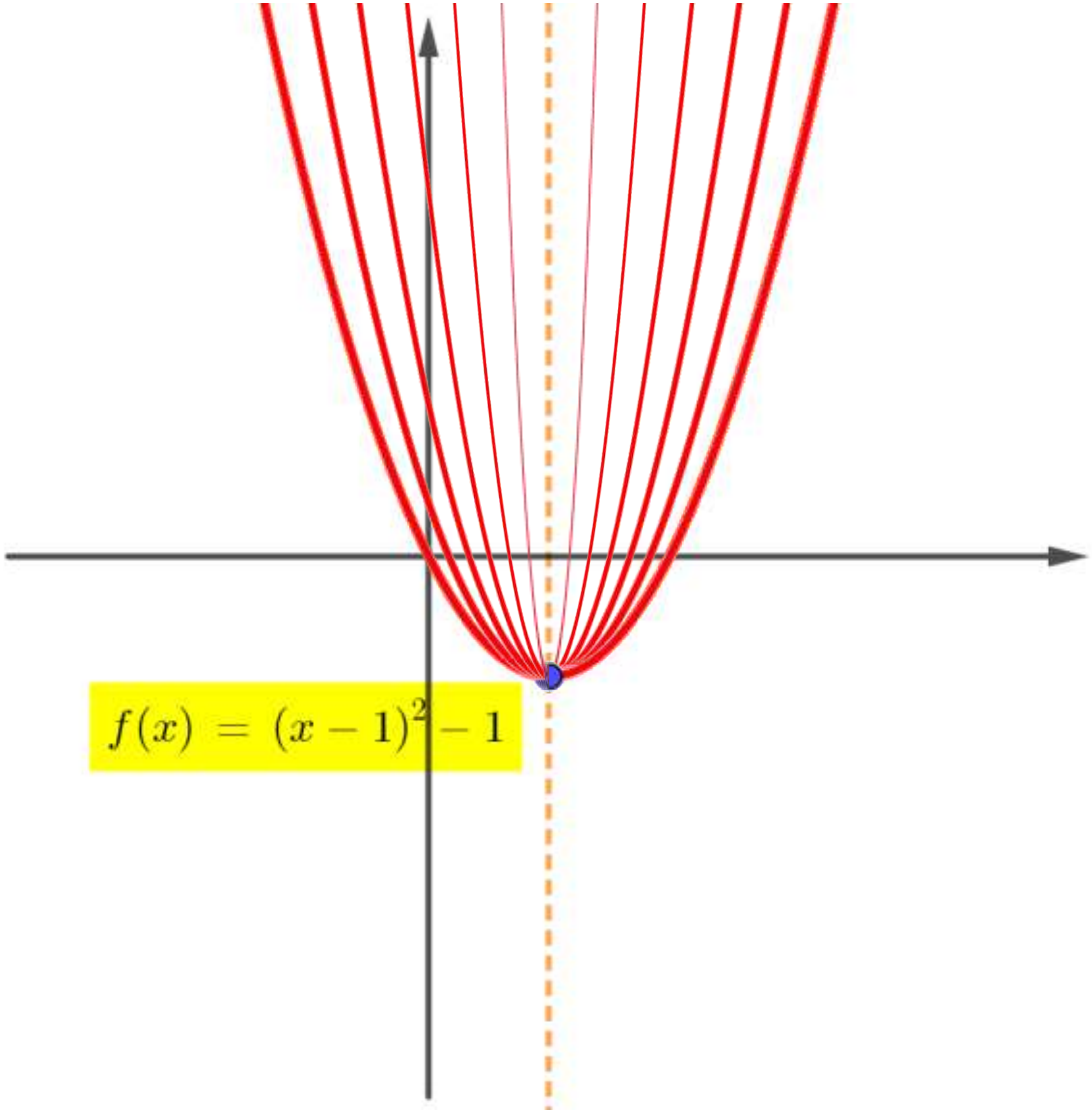


Odd function:
Domain is centered at O
O is the center of symmetry



Not even nor odd.
(y'y) is not an axis of symmetry nor O is a center of symmetry

Axis of symmetry



How to prove that a vertical line of equation $x = a$ is an axis of symmetry?

1 Domain is centered at a

$a - x \;\&\; a + x \in D_f$

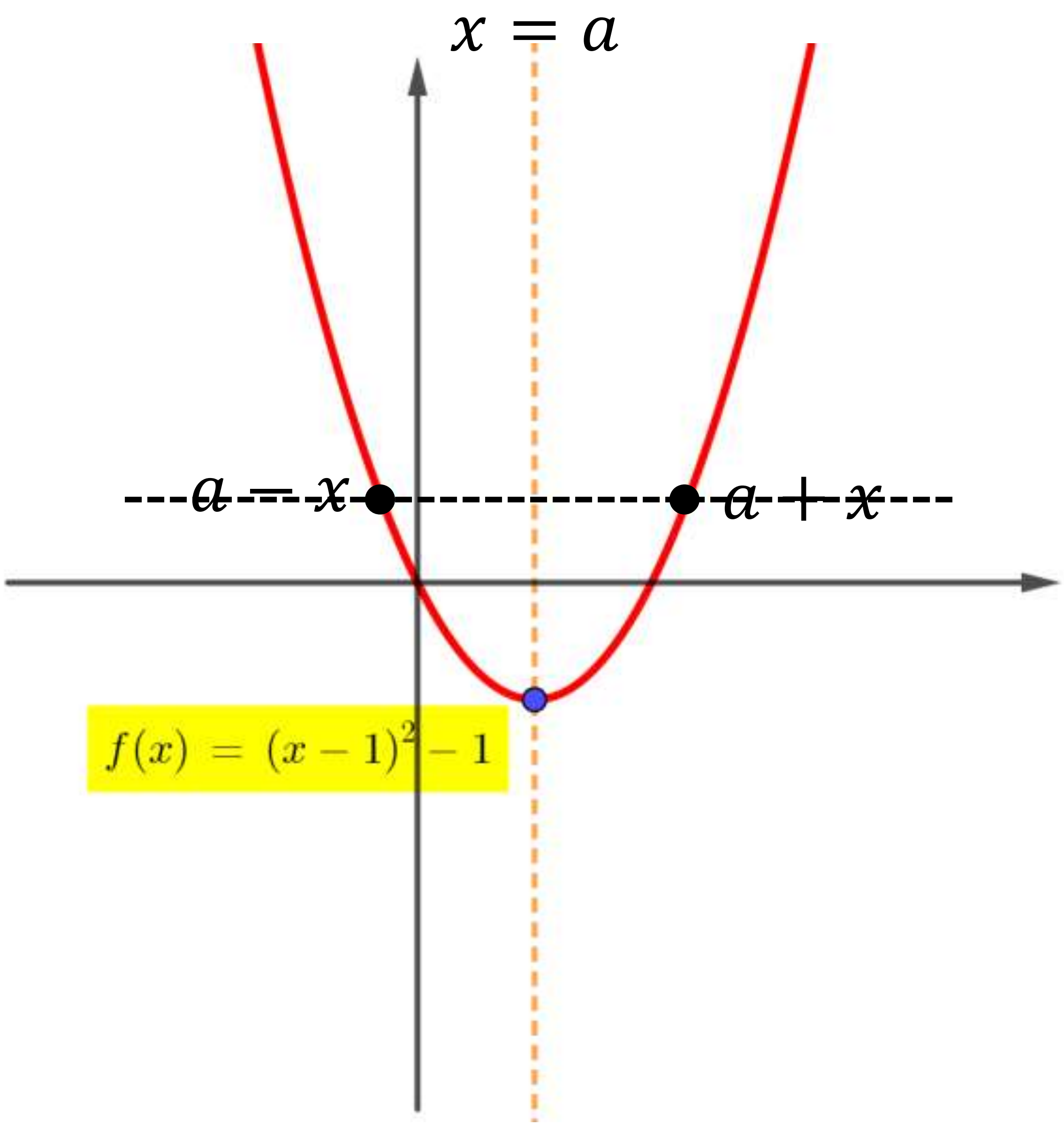
(this is not required to prove in grade 11)

2 $f(a-x) = f(a+x)$

or

$f(2a-x) = f(x)$

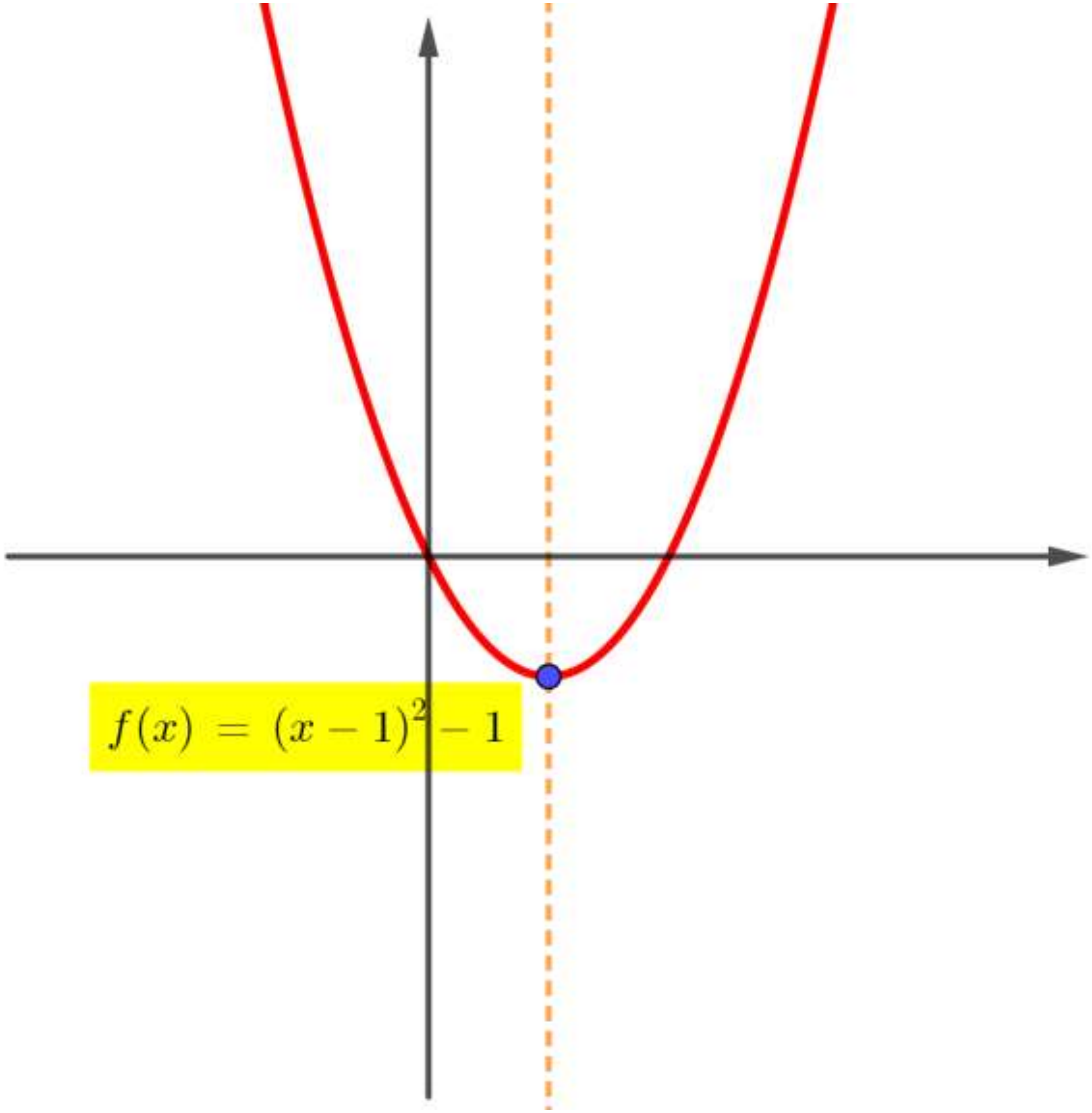
Axis of symmetry



Same ordinates

$f(a-x) = f(a+x)$

Axis of symmetry



Example:

$f(x) = (x-1)^2 - 1$

Show that the line of equation $x = 1$ is an axis of symmetry.

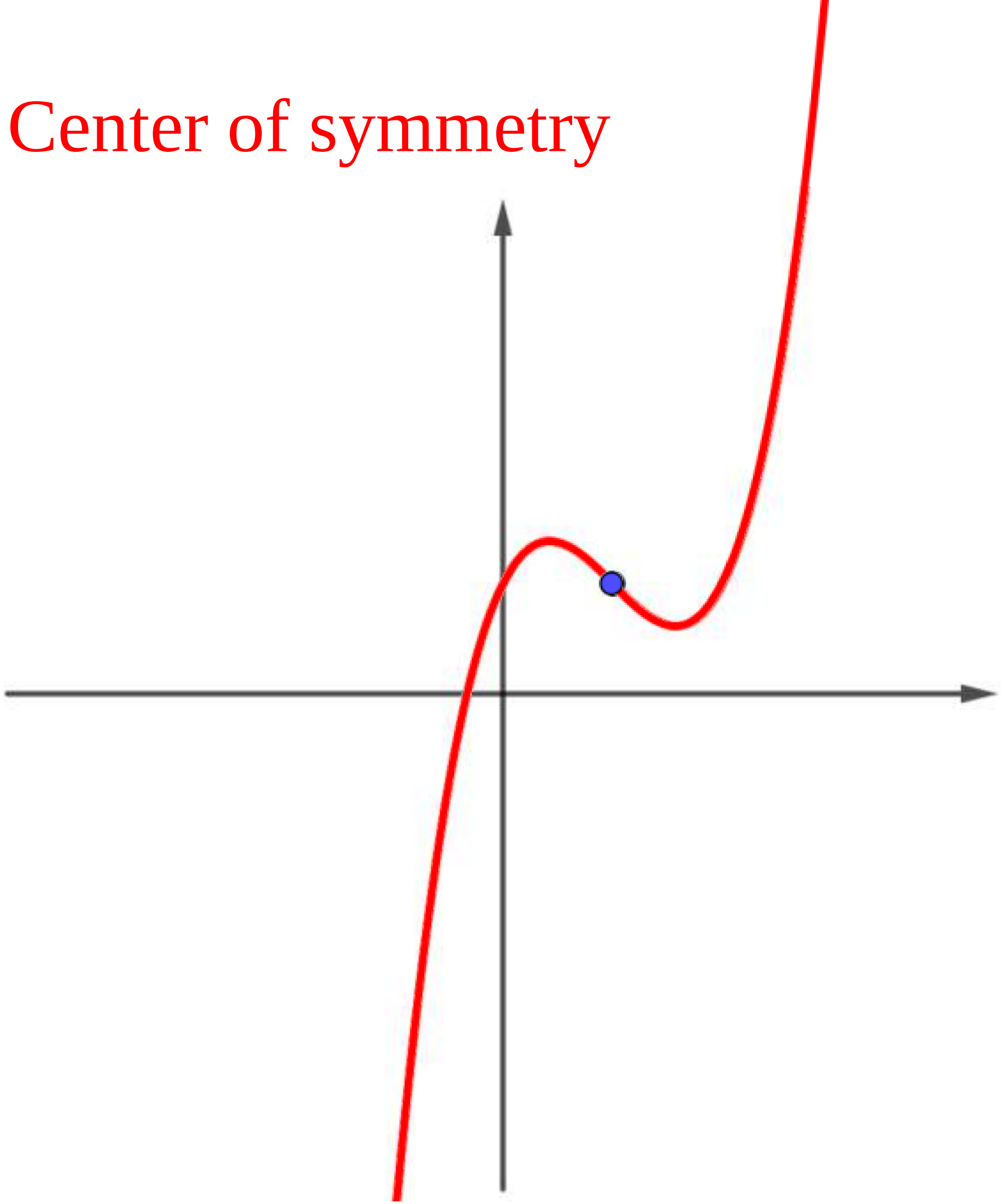
$D_f = IR =]-\infty; +\infty[$

Any real number is a center of IR

$$f(2a - x) = f(2(1) - x) = f(2 - x)$$
$$= (2 - x - 1)^2 - 1$$
$$= (1 - x)^2 - 1$$
$$= (x - 1)^2 - 1 = f(x)$$

So the line of equation $x = 1$ is an axis of symmetry.

Center of symmetry



How to prove that a point of coordinates (a, b) is a center of symmetry?

1 Domain is centered at a

$a - x \;\&\; a + x \epsilon D_f$

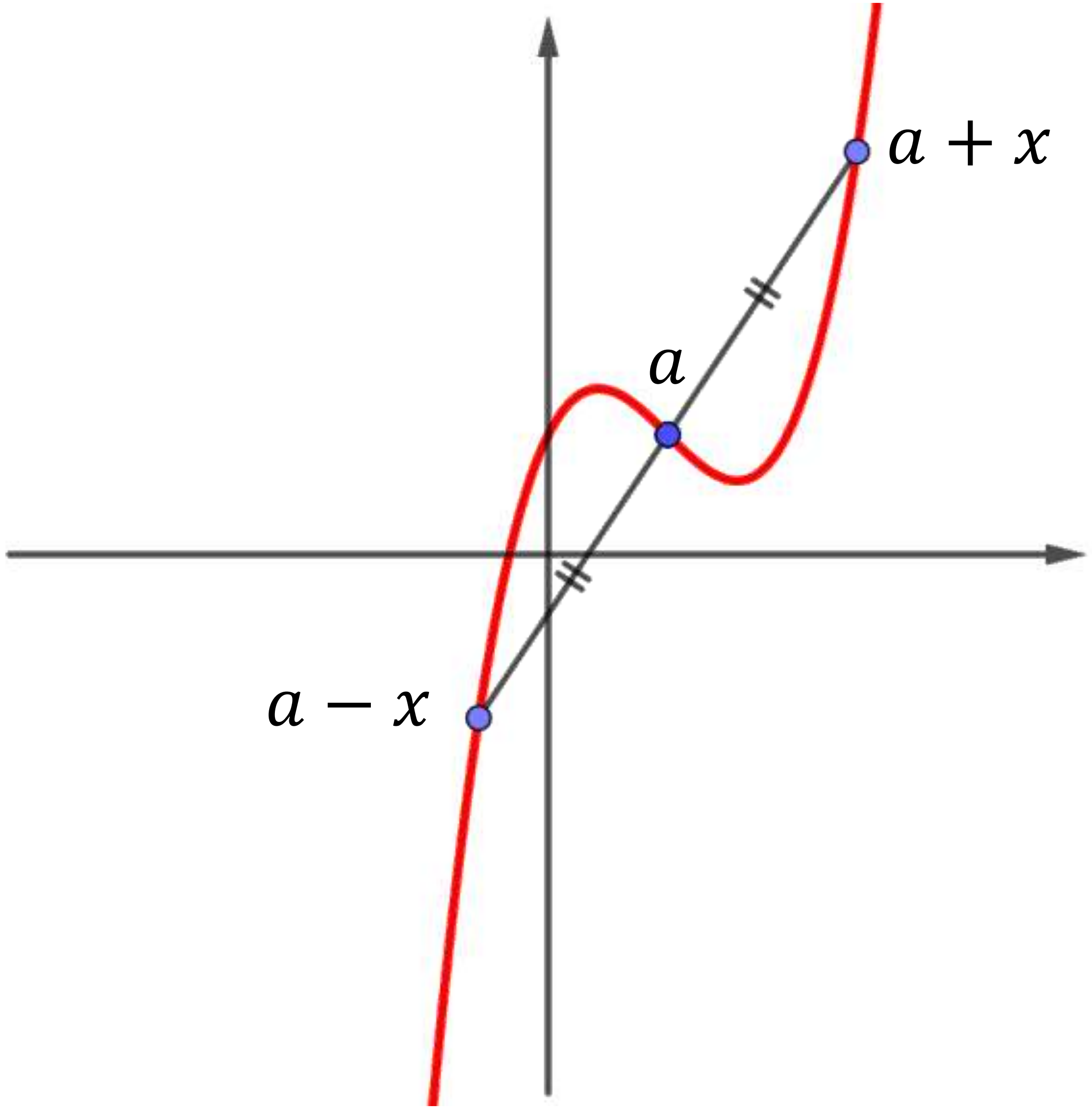
(this is not required to prove in grade 11)

2 $f(a - x) + f(a + x) = 2b$

or

$f(2a - x) + f(x) = 2b$

Center of symmetry



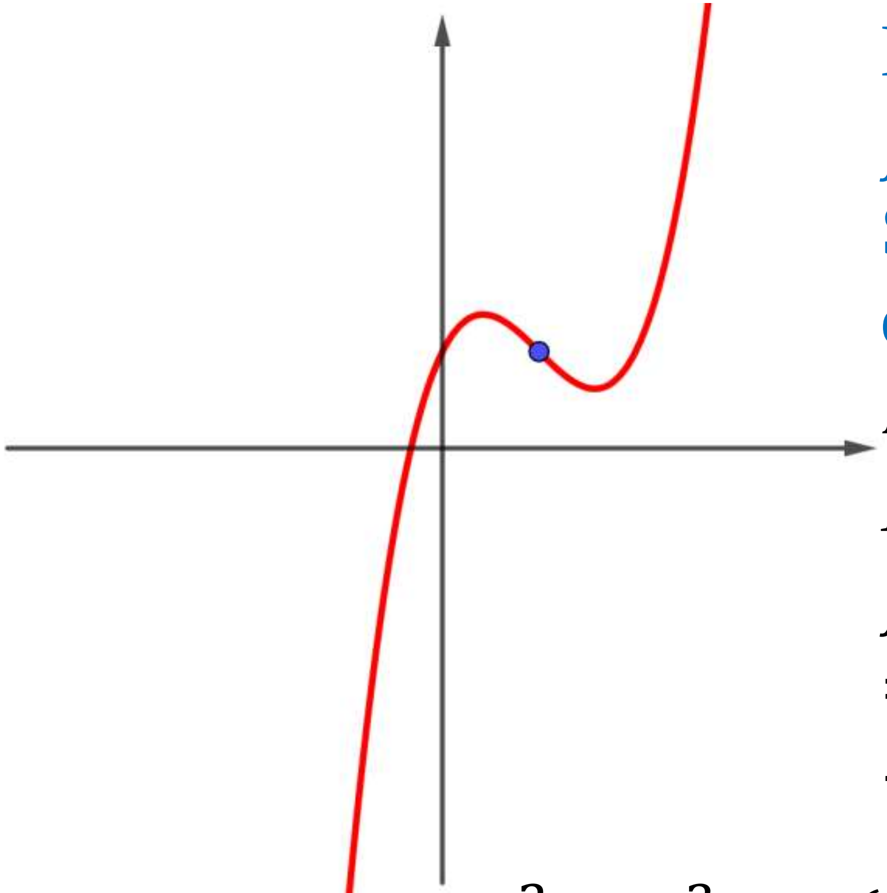
The point of abscissa a is the midpoint of the segment joining the points of abscissa $a-x$ and $a+x$

So $f(a)=\frac{f(a-x)+f(a+x)}{2}$

Then

$$f(a-x)+f(a+x)=2f(a)=2b$$

Center of symmetry



Example:

$f(x) = x^3 - 3x^2 + 2x + 1$

Show that the point of coordinates (1;1) is a center of symmetry.

$D_f = IR =]-\infty; +\infty[$

Any real number is a center of IR

$f(2a - x) + f(x) = f(2(1) - x) + f(x)$

$= f(2 - x) + f(x) = (2 - x)^3 - 3(2 - x)^2 + 2(2$

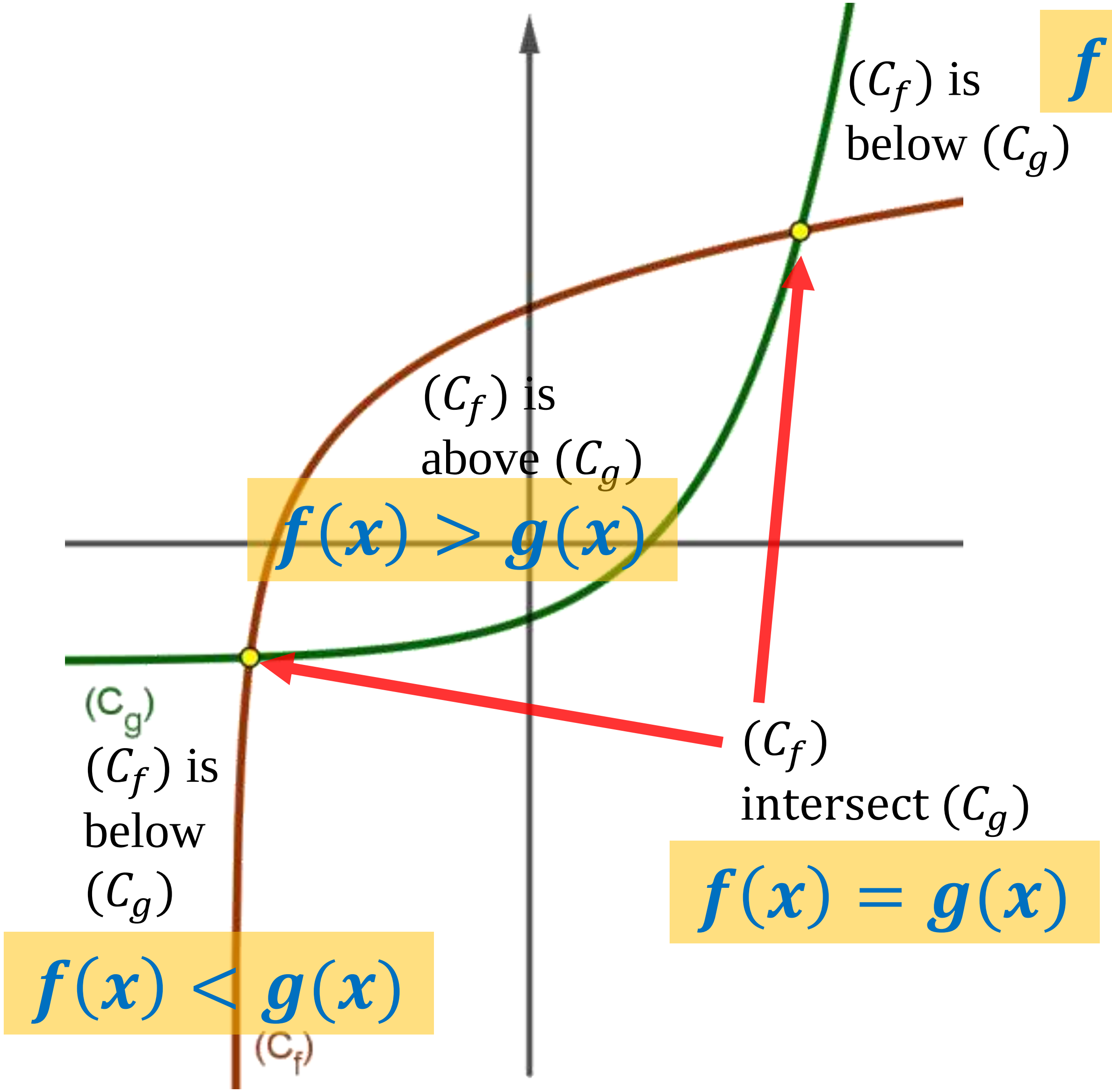
$- x) + 1 + x^3 - 3x^2 + 2x + 1$

$= 8 - 12x + 6x^2 - x^3 - 3(4 - 4x + x^2) + 4 - 2x + 1 + x^3 - 3x^2 + 2x + 1$

$= 14 - 12x + 3x^2 - 12 + 12x - 3x^2 = 2 = 2(1)$

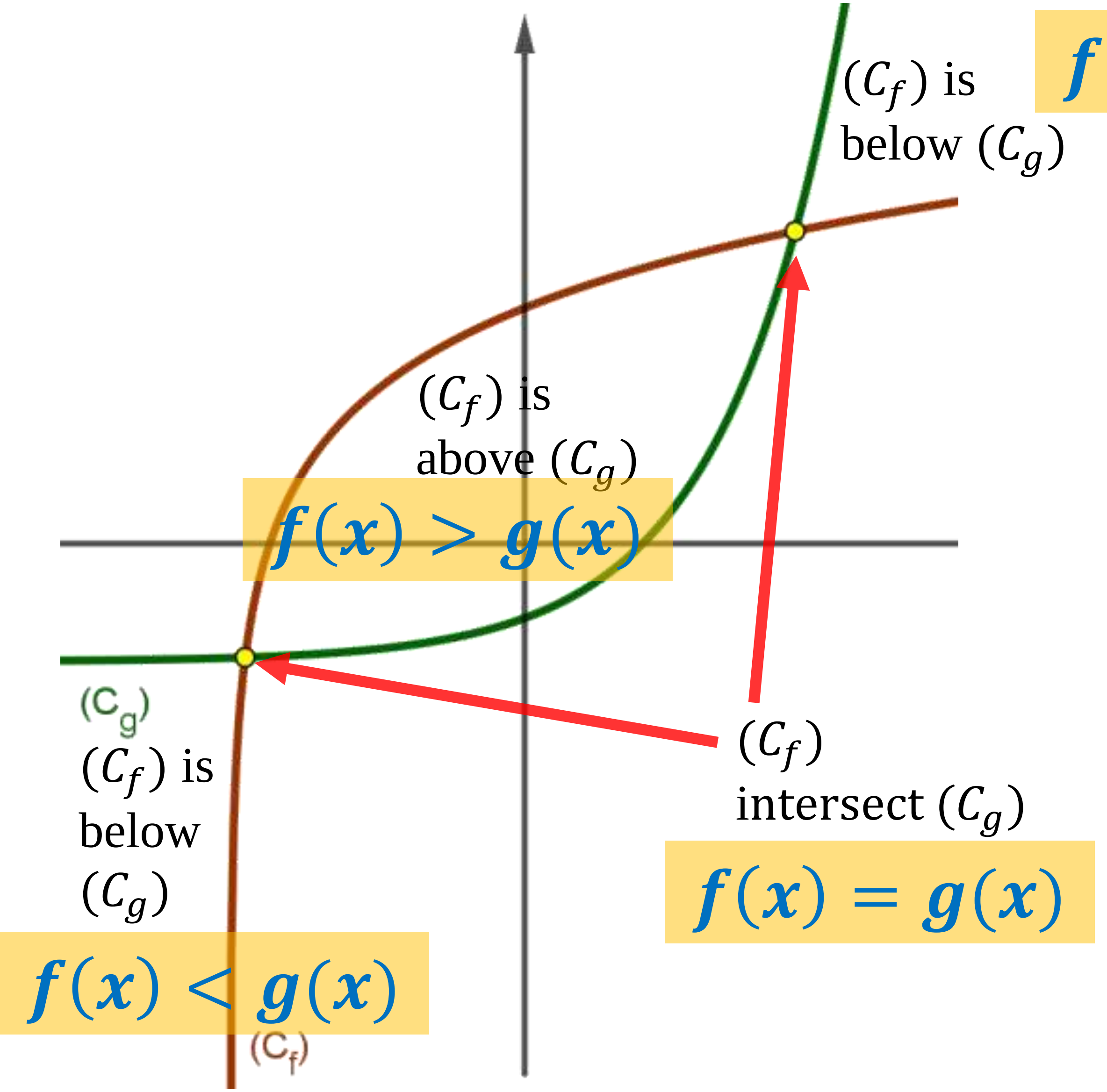
So the point of coordinates (1;1) is a center of symmetry.

Relative position between two curves



HOW?

Relative position between two curves



HOW?

Step 1: Calculate $f(x) - g(x)$

Step 2: Study the sign of $f(x) - g(x)$

Step 3: Interpret

- If $f(x) - g(x) > 0$

  (C_f) is **above** (C_g)
- If $f(x) - g(x) < 0$

  (C_f) is **below** (C_g)
- If $f(x) - g(x) = 0$

  (C_f) **intersects** (C_g)

Relative position between two curves

Example:

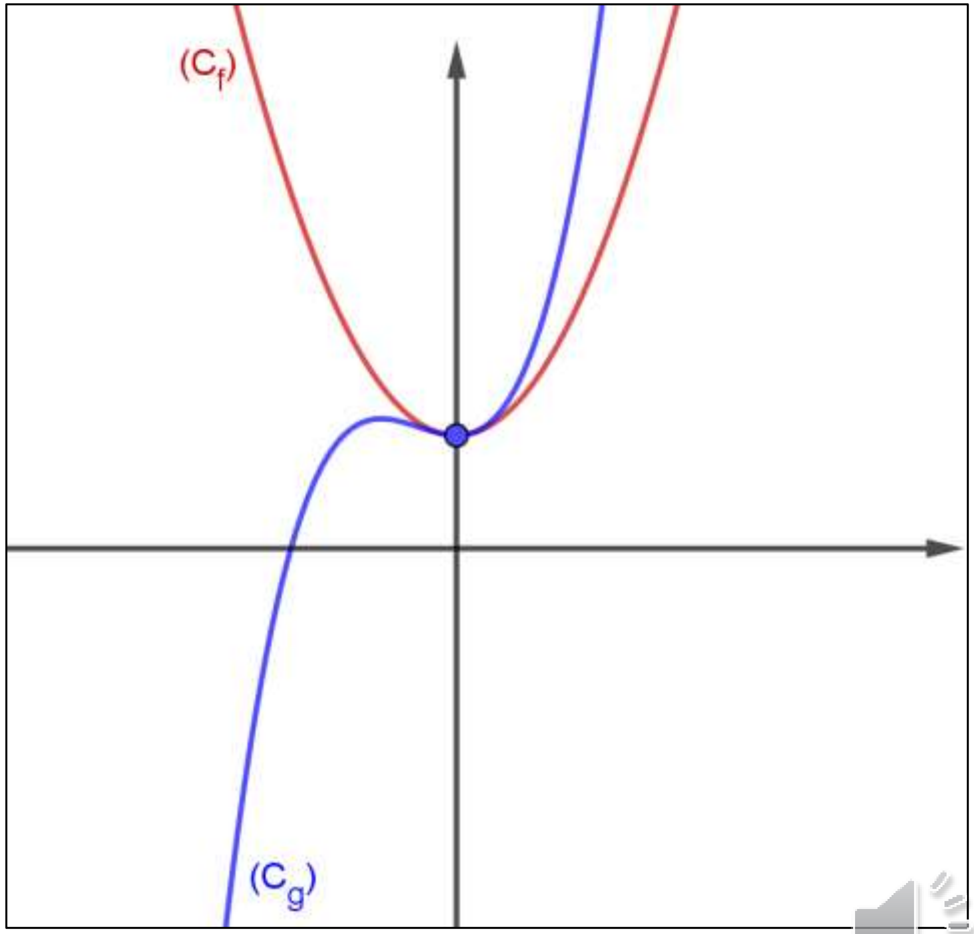
Study the relative position between the curves of the functions:

$f(x) = x^2 + 1$ and $g(x) = x^3 + x^2 + 1$

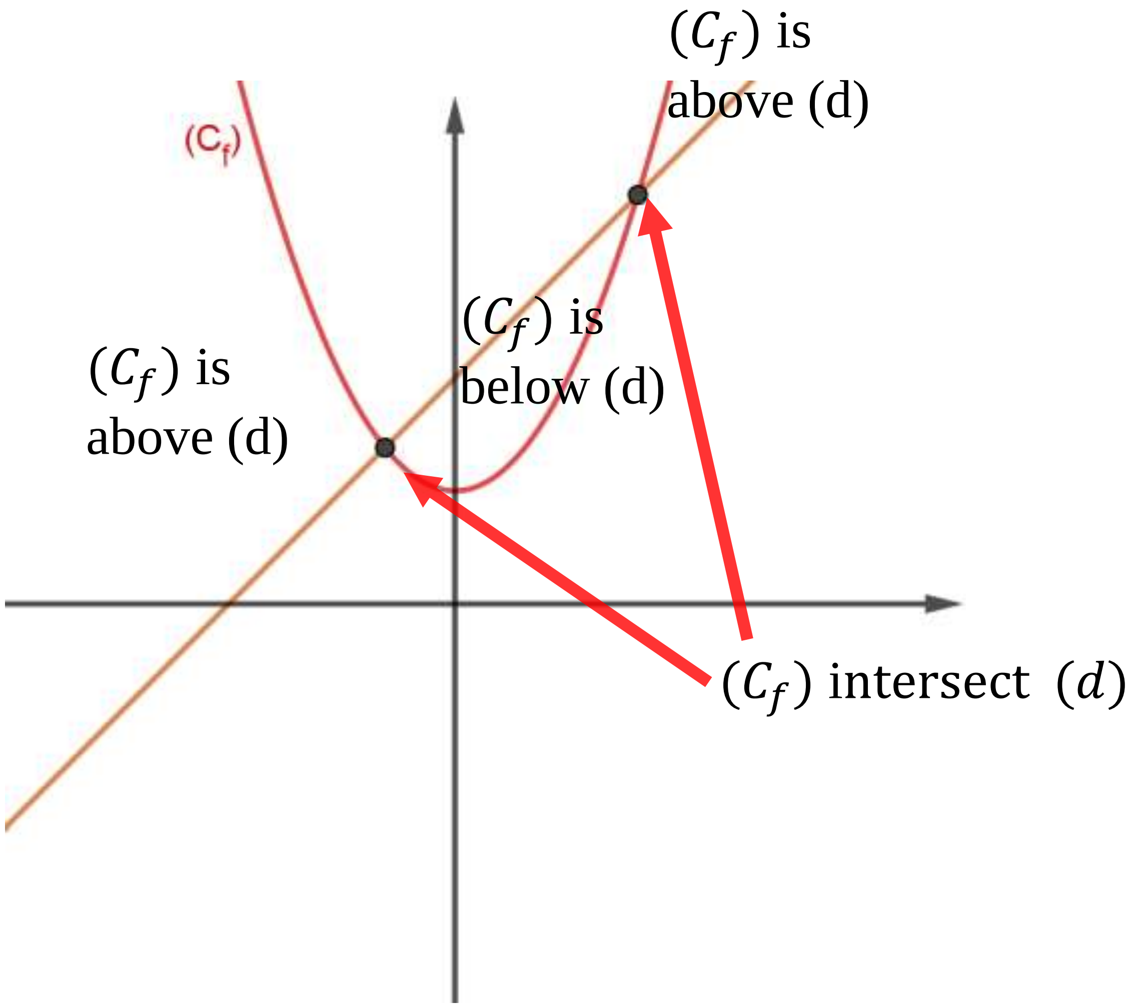
$$f(x) - g(x) = x^2 + 1 - x^3 - x^2 - 1 = -x^3$$

x		0	
$f(x) - g(x)$	+	0	−
Position between (C_f) and (C_g)	(C_f) **above** (C_g)		(C_f) **below** (C_g)

(C_f) intersect (C_g) at (0,1)



Relative position between a curve and a line of equation $y = ax + b$



Same as between two curves

Step 1: Calculate $f(x) - y$

Step 2: Study the sign of $f(x) - y$

Step 3: Interpret

- If $f(x) - y > 0$
  (C_f) is **above** (d)
- If $f(x) - y < 0$
  (C_f) is **below** (d)
- If $f(x) - y = 0$
  (C_f) **intersects** (d)

Relative position between two curves

Example:

Study the relative position between the curves of the functions:

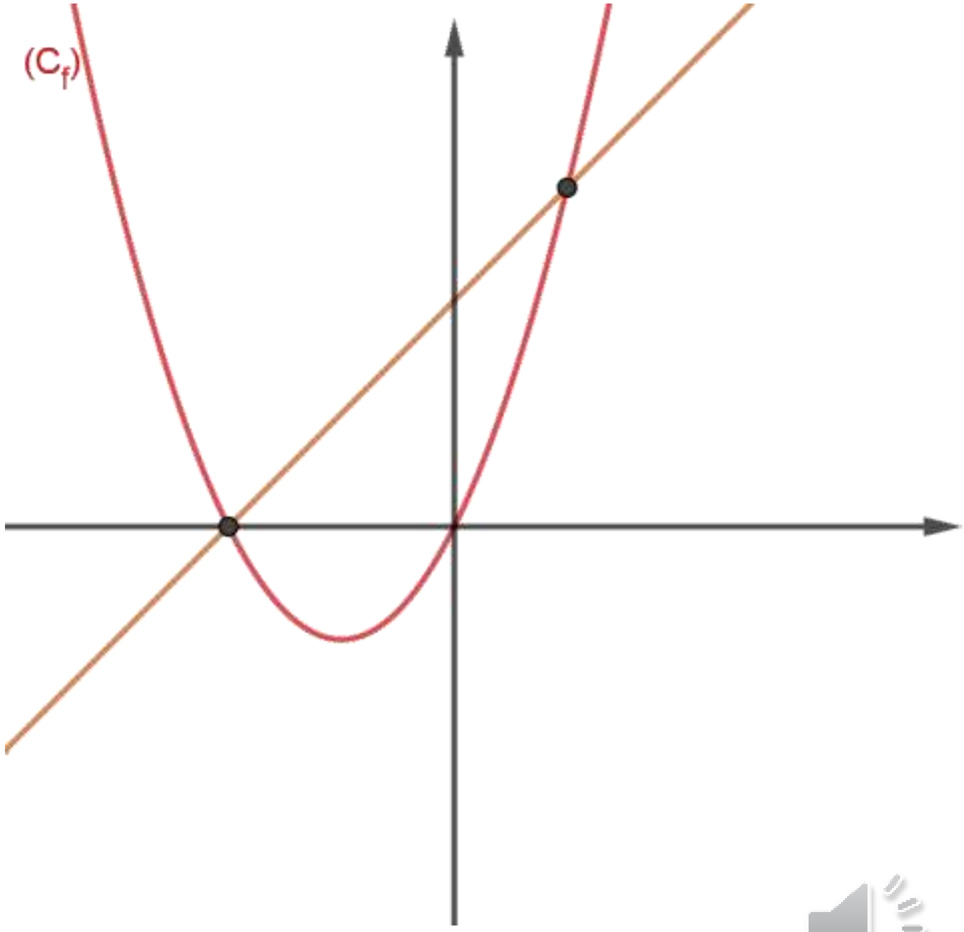
$f(x) = x^2 + 2x$ and (d) : $y = x + 2$

$$f(x) - y = x^2 + 2x - x - 2 = x^2 + x - 2$$

x		0		1	
$f(x) - g(x)$	$+$	0	$-$	0	$+$
Position between (C_f) and (C_g)	(C_f) above (d)		(C_f) below (d)		(C_f) above (d)

(C_f) cuts (d) at (0,2)

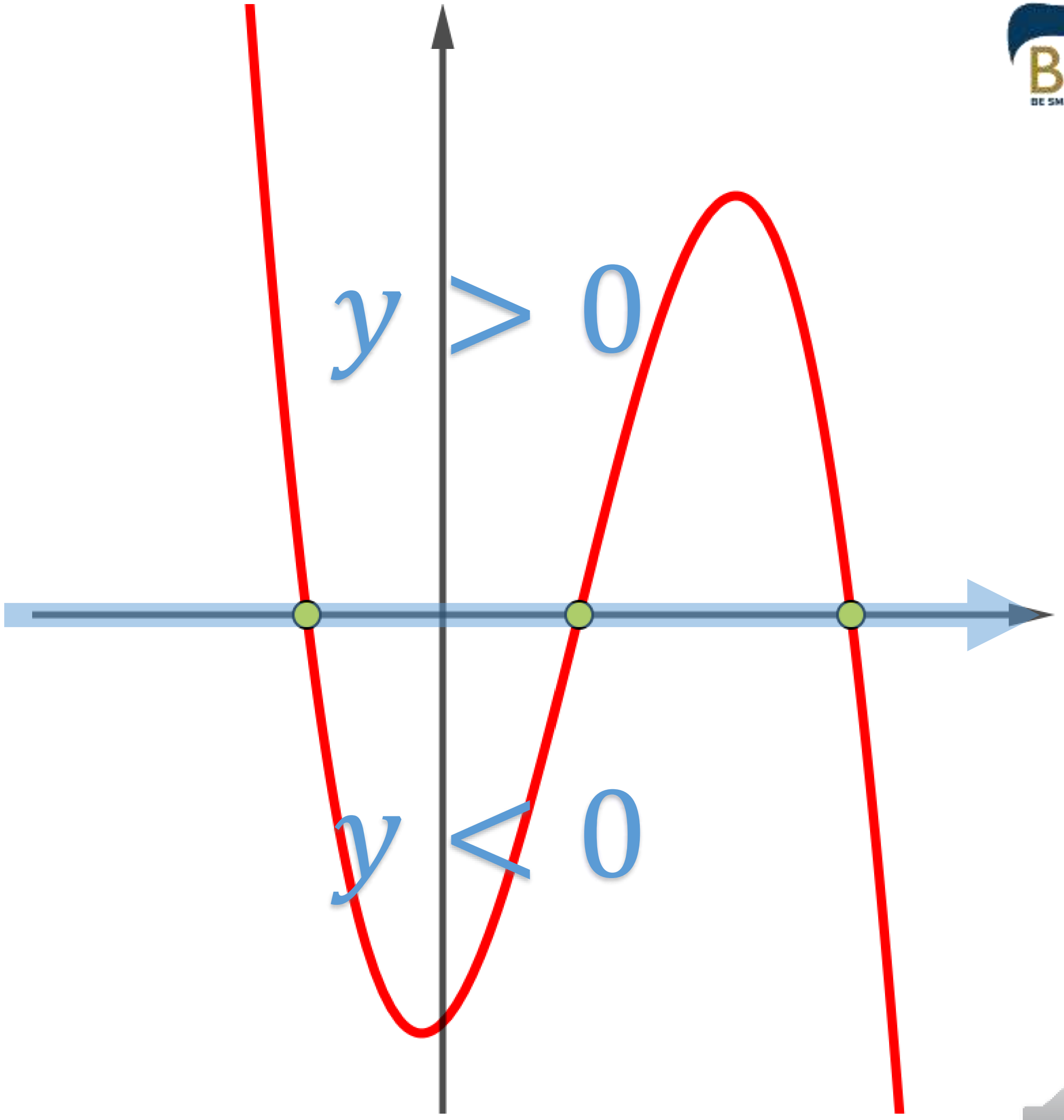
(C_f) cuts (d) at (1,3)



Sign of a function (graphically)

The sign of the function, graphically, is determined according to its position with respect to (x'x).

WHY (X'X)?

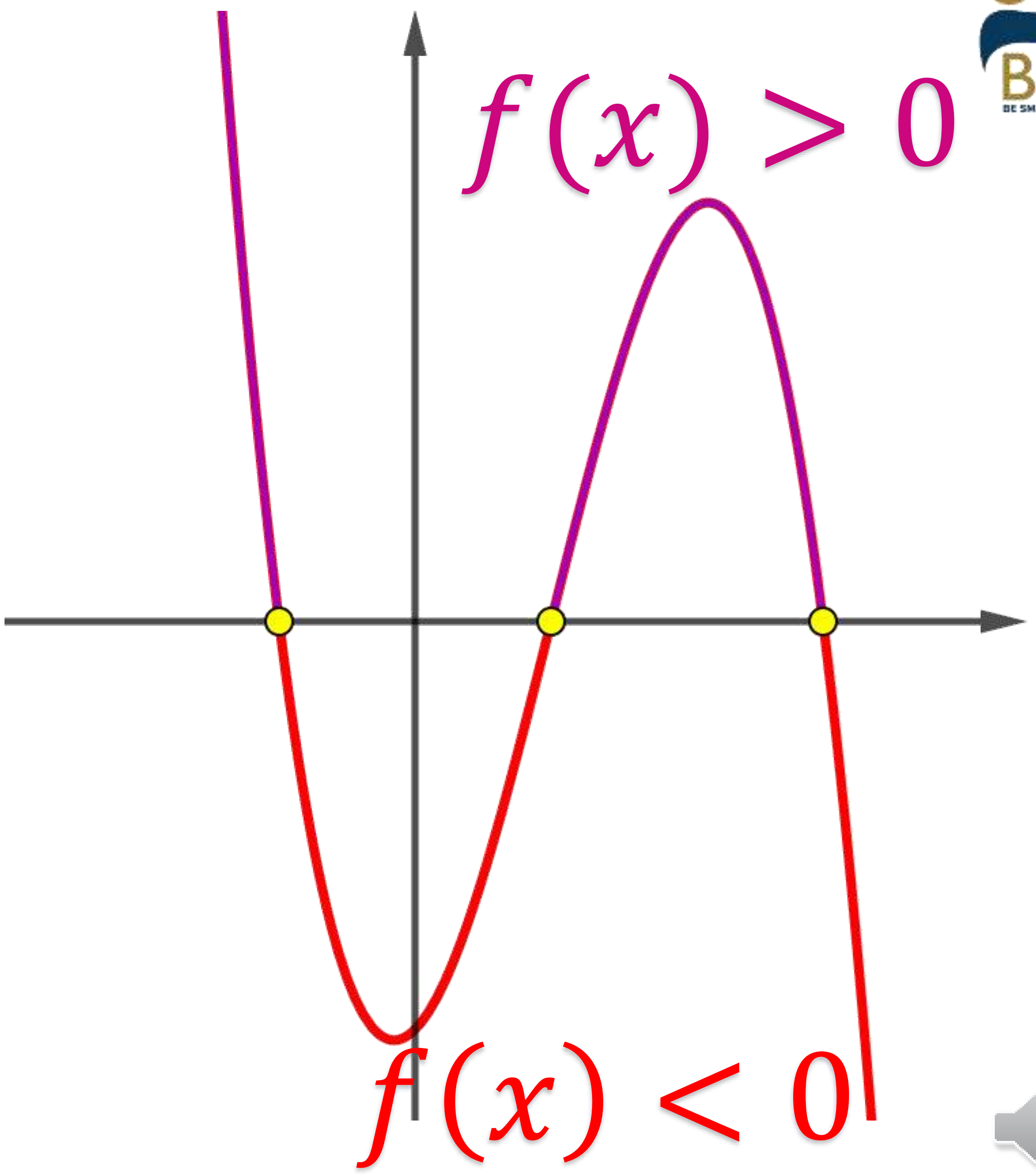


Sign of a function (graphically)

The sign of the function, graphically, is determined according to its position with respect to (x'x).

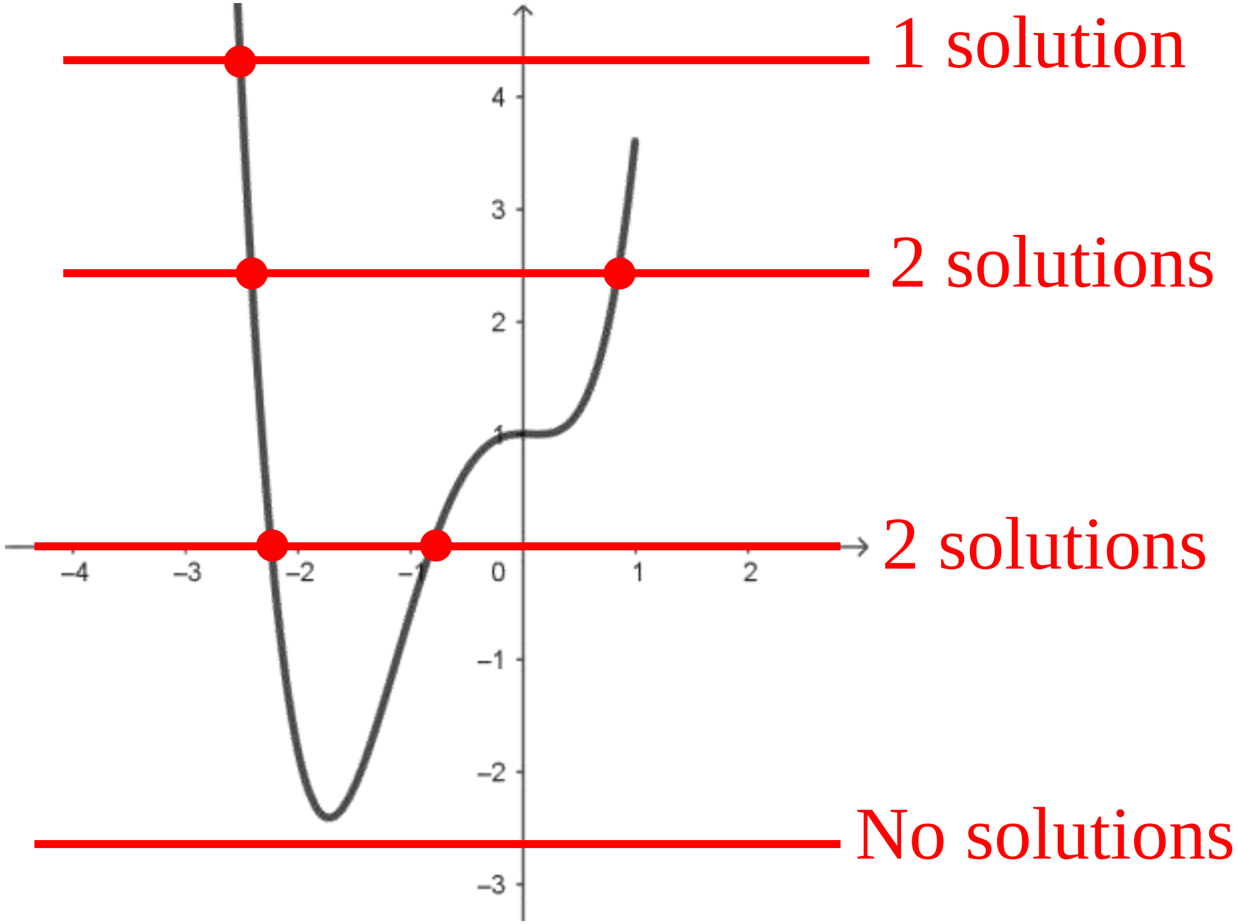
In this example:

- $f(x) > 0 : x \in]-\infty; -1[\cup]1; 3[$
- $f(x) < 0 : x \in]-1; 1[\cup]3; +\infty[$
- $f(x) = 0 : x \in \{-1; 1; 3\}$



Solving $f(x) = k$ (graphically)

Consider the function f of representative curve (C_f).
To solve graphically $f(x) = k$ it is sufficient to find the intersecting points between (C_f) and the horizontal line of equation $y = k$.



Solving $f(x) = k$ (graphically)

Example:

Consider the function f defined over IR and of representative curve (C_f).

Solve :

1) $f(x) = 1$

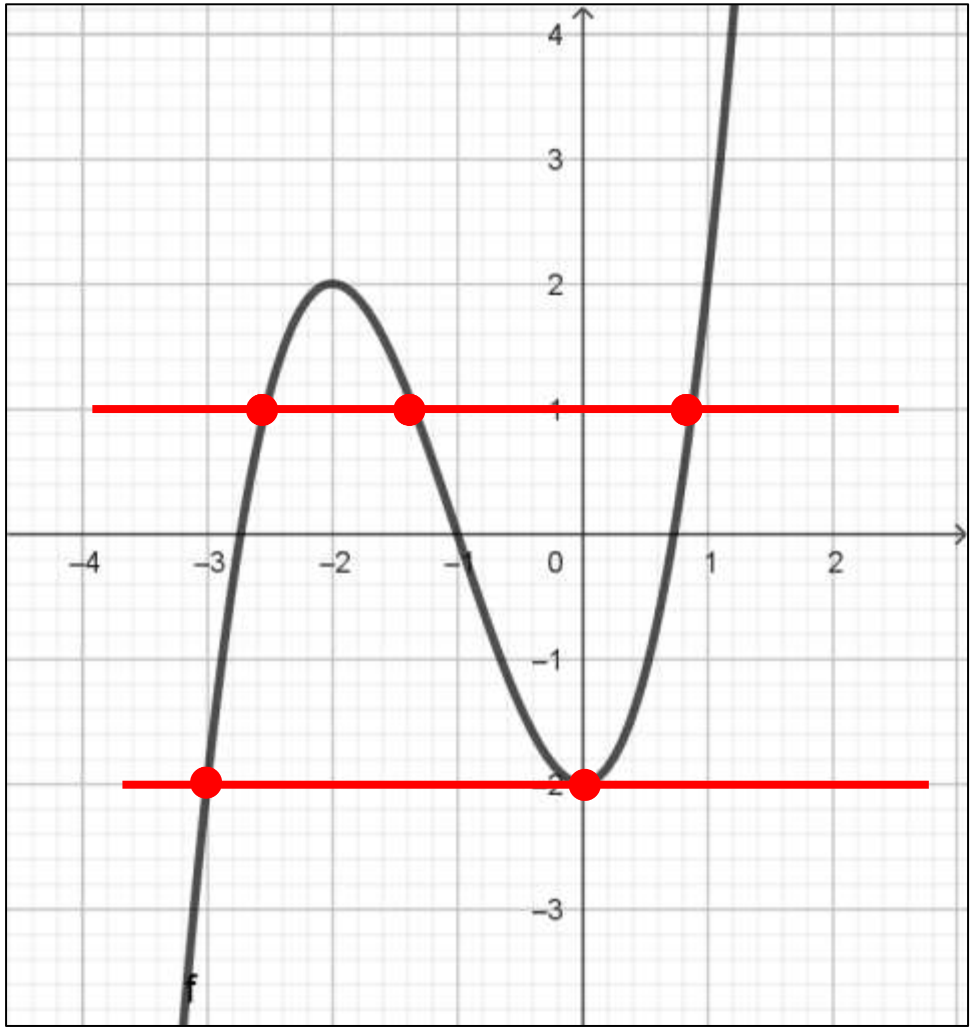
3 solutions

2) $f(x) = -2$

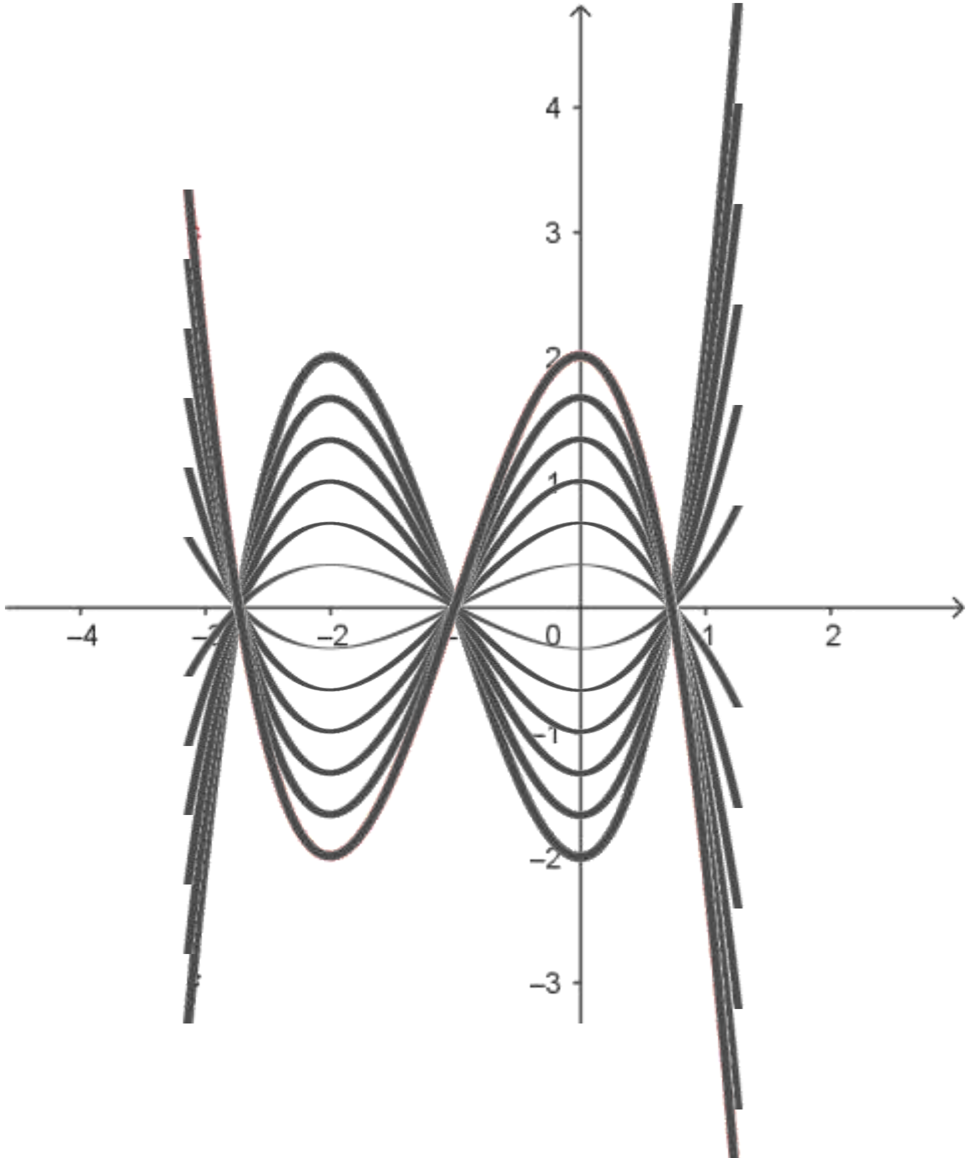
2 points of intersection

one double solution

One single

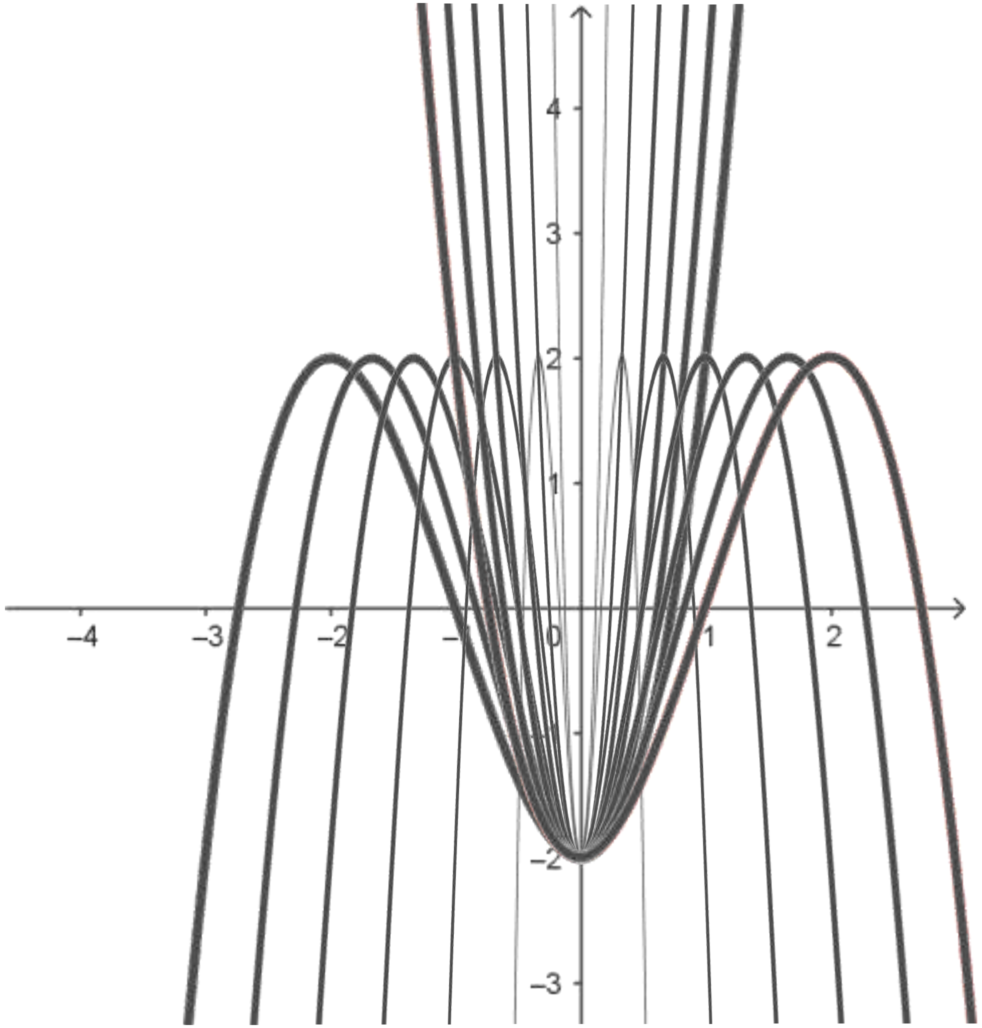


Reflection with respect to (x’x)



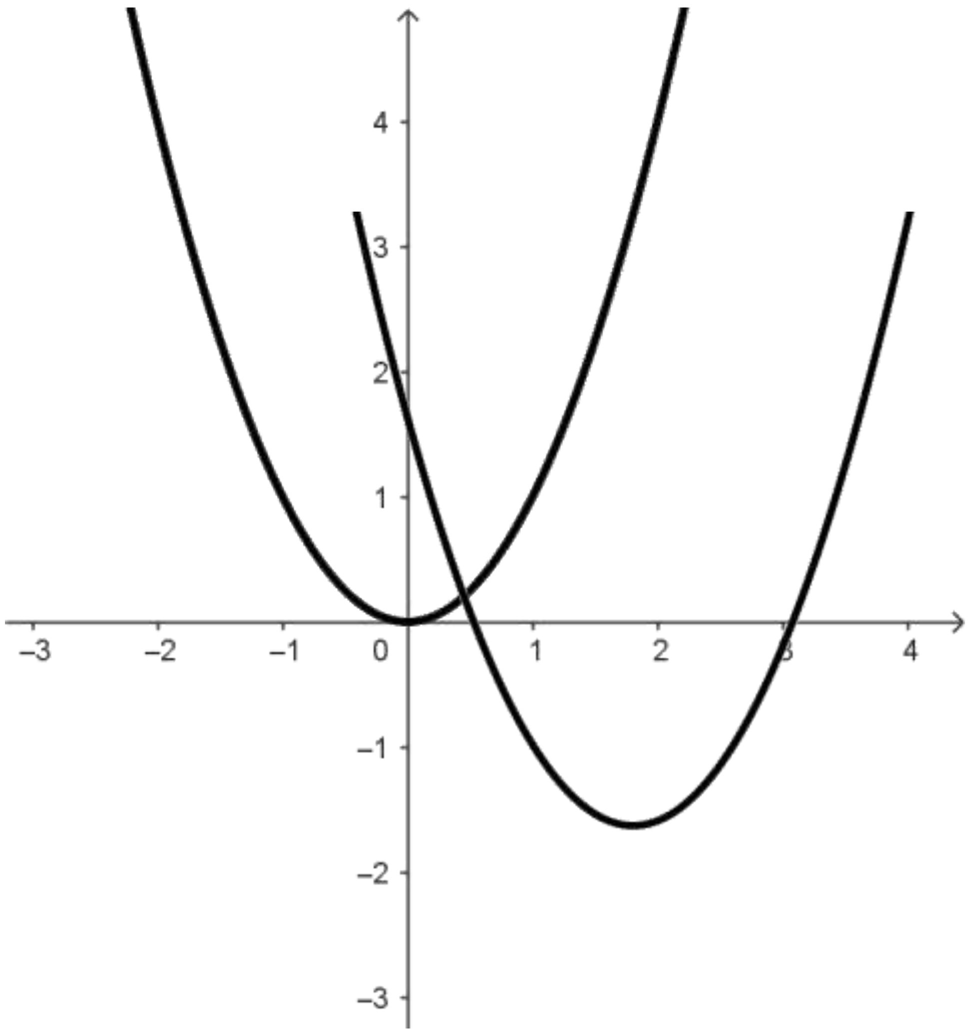
(C_f) and (C_g) are symmetric with respect to (x’x): $g(x) = -f(x)$

Reflection with respect to (y'y)



(C_f) and (C_g) are symmetric with respect to (y'y): $g(x) = f(-x)$

Translation



$(C_g)$is the translate of (C_f) by the translation of vector (a, b):

$g(x) = f(x - a) + b$